General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MPC4

Unit Pure Core 4

Thursday 16 June 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 13x + 6$.

(a) Find $f(-2)$. *(1 mark)*

(b) Use the Factor Theorem to show that $2x - 3$ is a factor of $f(x)$. *(2 marks)*

(c) Simplify $\dfrac{2x^2 + x - 6}{f(x)}$. *(4 marks)*

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2 The average weekly pay of a footballer at a certain club was £80 on 1 August 1960. By 1 August 1985, this had risen to £2000.

The average weekly pay of a footballer at this club can be modelled by the equation

$$P = Ak^t$$

where £P is the average weekly pay t years after 1 August 1960, and A and k are constants.

(a) (i) Write down the value of A. *(1 mark)*

(ii) Show that the value of k is 1.137411, correct to six decimal places. *(2 marks)*

(b) Use this model to predict the year in which, on 1 August, the average weekly pay of a footballer at this club will first exceed £100 000. *(3 marks)*

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3 (a) (i) Find the binomial expansion of $(1 - x)^{\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$(125 - 27x)^{\frac{1}{3}} \approx 5 + \frac{m}{25}x + \frac{n}{3125}x^2$$

for small values of x, stating the values of the integers m and n. *(3 marks)*

(b) Use your result from part **(a)(ii)** to find an approximate value of $\sqrt[3]{119}$, giving your answer to five decimal places. *(2 marks)*

4 (a) A curve is defined by the parametric equations $x = 3\cos 2\theta$, $y = 2\cos\theta$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{k\cos\theta}$, where k is an integer. *(4 marks)*

(ii) Find an equation of the normal to the curve at the point where $\theta = \dfrac{\pi}{3}$. *(4 marks)*

(b) Find the exact value of $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \,\mathrm{d}x$. *(5 marks)*

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5 The points A and B have coordinates $(5,\ 1,\ -2)$ and $(4,\ -1,\ 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} -8 \\ 5 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$.

(a) Find a vector equation of the line that passes through A and B. *(3 marks)*

(b) (i) Show that the line that passes through A and B intersects the line l, and find the coordinates of the point of intersection, P. *(4 marks)*

(ii) The point C lies on l such that triangle PBC has a right angle at B. Find the coordinates of C. *(5 marks)*

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6 A curve is defined by the equation $2y + \mathrm{e}^{2x}y^2 = x^2 + C$, where C is a constant. The point $P\left(1,\ \dfrac{1}{\mathrm{e}}\right)$ lies on the curve.

(a) Find the exact value of C. *(1 mark)*

(b) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. *(7 marks)*

(c) Verify that $P\left(1,\ \dfrac{1}{\mathrm{e}}\right)$ is a stationary point on the curve. *(2 marks)*

7 A giant snowball is melting. The snowball can be modelled as a sphere whose surface area is decreasing at a constant rate with respect to time. The surface area of the sphere is $A\,\text{cm}^2$ at time t days after it begins to melt.

(a) Write down a differential equation in terms of the variables A and t and a constant k, where $k > 0$, to model the melting snowball. *(2 marks)*

(b) (i) Initially, the radius of the snowball is 60 cm, and 9 days later, the radius has halved.

Show that $A = 1200\pi(12 - t)$.

(You may assume that the surface area of a sphere is given by $A = 4\pi r^2$, where r is the radius.) *(4 marks)*

(ii) Use this model to find the number of days that it takes the snowball to melt completely. *(1 mark)*

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8 (a) Express $\dfrac{1}{(3-2x)(1-x)^2}$ in the form $\dfrac{A}{3-2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$. *(4 marks)*

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{y}}{(3-2x)(1-x)^2}$$

where $y = 0$ when $x = 0$, expressing your answer in the form

$$y^p = q\ln[\mathrm{f}(x)] + \frac{x}{1-x}$$

where p and q are constants. *(9 marks)*

END OF QUESTIONS